Chapter 5
Thrust/ Weight (T/W) and Wing Loading (W/S)

Thrust/Weight and Wing Loading

Unless specified otherwise thrust/weight and wing loading refer to reference conditions (below). If there is any doubt the suffix$_{ref}$ should be appended

$$\textbf{Thrust/Weight Ratio} = \left(\frac{T}{W}\right)_{ref}$$

where T = Maximum dry thrust with all engines running at sea level, static conditions, installed

W = Maximum takeoff gross weight at specified condition

(usually block release, but maybe brake release at start of takeoff roll)

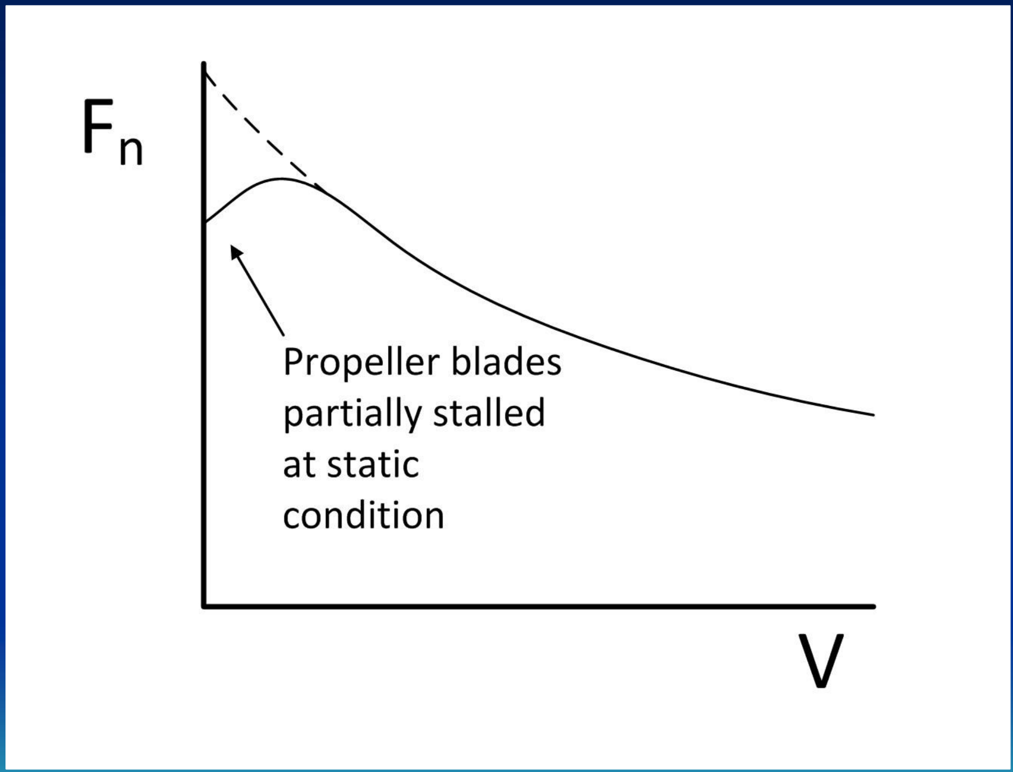
$$\textbf{Wing Loading} = \left(\frac{W}{S}\right)_{ref}$$

where S = Reference wing area

Why $\frac{W}{S}$ ***and not*** $\frac{S}{W}$ ***? Because*** $\frac{W}{S}$ ***relates to the lift coefficient*** $C_L = \frac{1}{q}\left(\frac{W}{S}\right)$ ***and has units of pressure***

Thrust versus Speed Lapse Rate

Anomalous $(T/W)_{ref}$ results obtained if thrust lapse rate with speed is also anomalous



T/W and W/S related to Payload-Range and Performance

- Engines and wing should be no larger than they have to be
  - T/W should be as low as possible
  - W/S should be as high as possible
- while still meeting all performance requirements

- Takeoff field length
- Landing field length
- Approach speed
- Initial climb
- Enroute climb
- Initial cruise altitude
- Sustained turn
- Instantaneous turn
- Acceleration
- Specific excess power (SEP)

Performance Constraints

- Air maneuvers
  - Steady level flight**
  - Steady climb**
  - Level acceleration*
  - Sustained turn*
  - Specific excess power*
- All the above can be determined using one equation

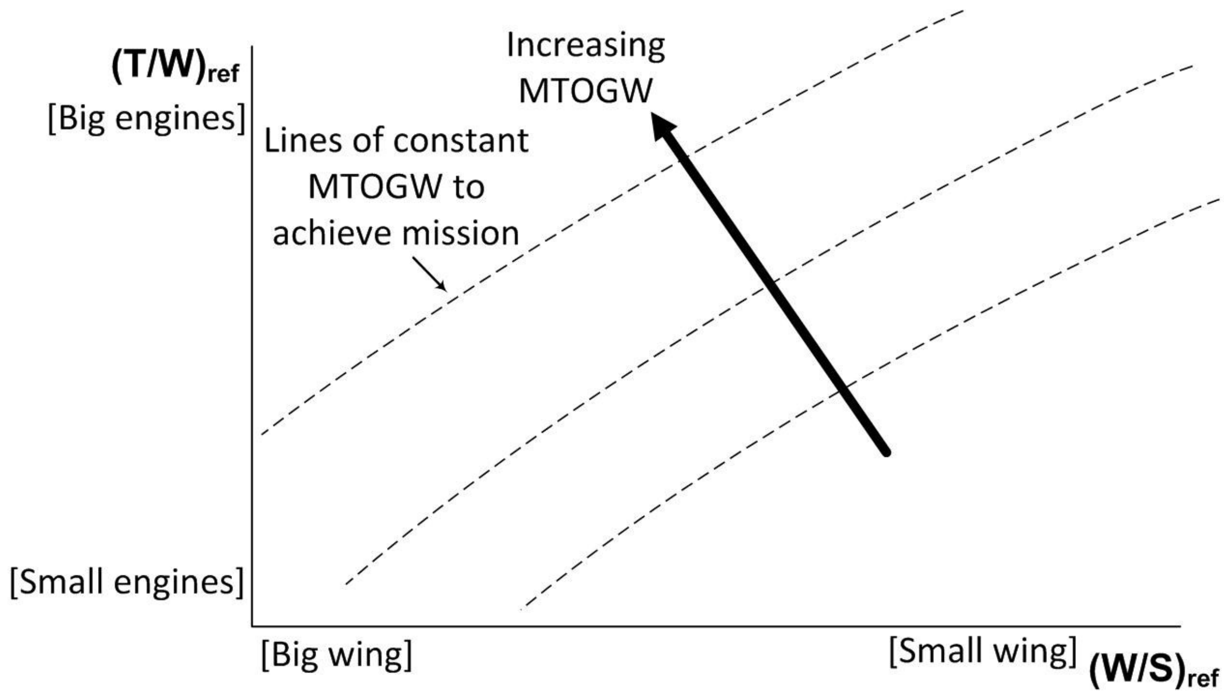
- Other constraints
  - Takeoff **
  - Approach speed*
  - Landing distance*
  - Second segment climb*
  - Instantaneous turn *

* Covered in Ch 17 Performance

** Covered in this chapter and in Ch 17 Performance

Mission Sizing

Fly mission for different values of T/W and W/S



Raymer:
Ch 3 Sizing from a Sketch
Ch 5 Initial Sizing
Ch 19 Trade Studies

Factoring Values to Reference Conditions

For equations in the form

$$T/W = f(W/S, var_1, var_2, \text{etc})$$

2

Factor *output* value of $\frac{T}{W}$

$$\left(\frac{T}{W}\right)_{ref} = \frac{T}{W}\frac{W}{W_{ref}}\frac{T_{ref}}{T}$$

where

$T =$ thrust at that performance condition

$T_{ref} =$ reference thrust (usually sea level static installed thrust at takeoff rating with all engines operating)

Obtain $\frac{T_{ref}}{T}$ from engine performance deck

1

Factor *input* value of $\frac{W_{ref}}{S}$

$$\frac{W}{S} = \frac{W_{ref}}{S}\frac{W}{W_{ref}}$$

where

$W =$ weight at that performance condition

$W_{ref} =$ max TOGW

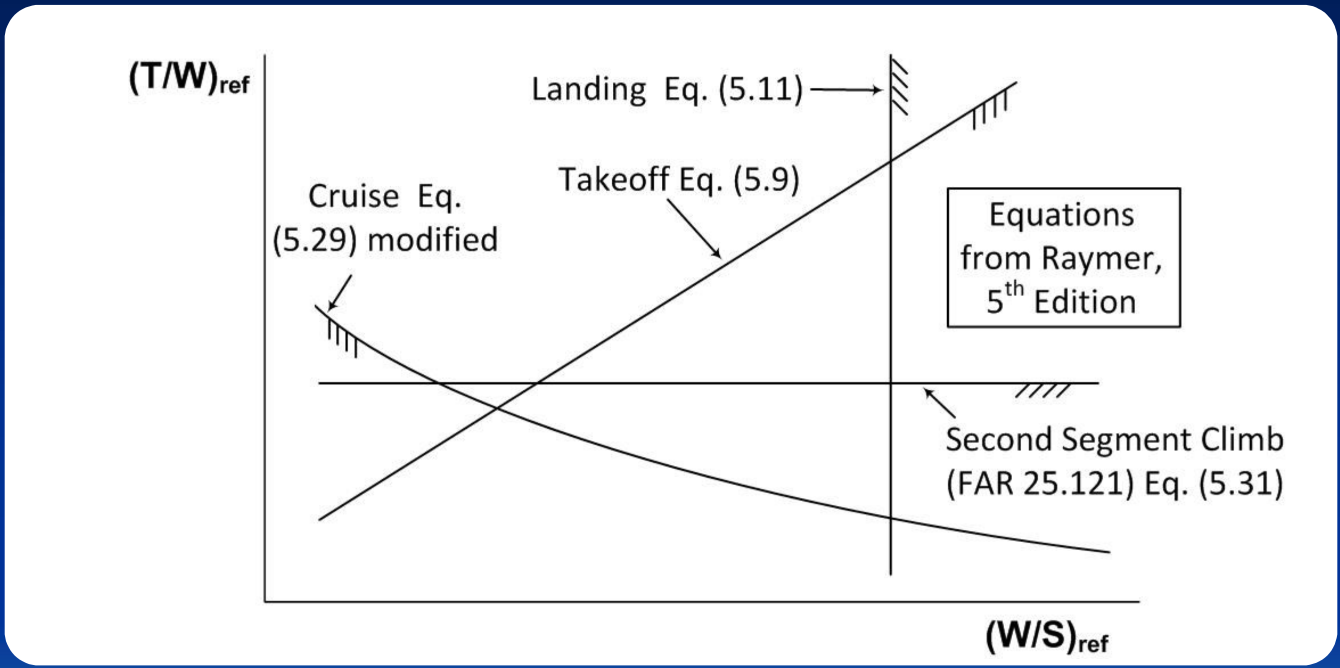
$S =$ reference wing area

Obtain $\frac{W}{W_{ref}}$ from mission routine

Sizing and Performance – Commercial

Use these equations for student exercise

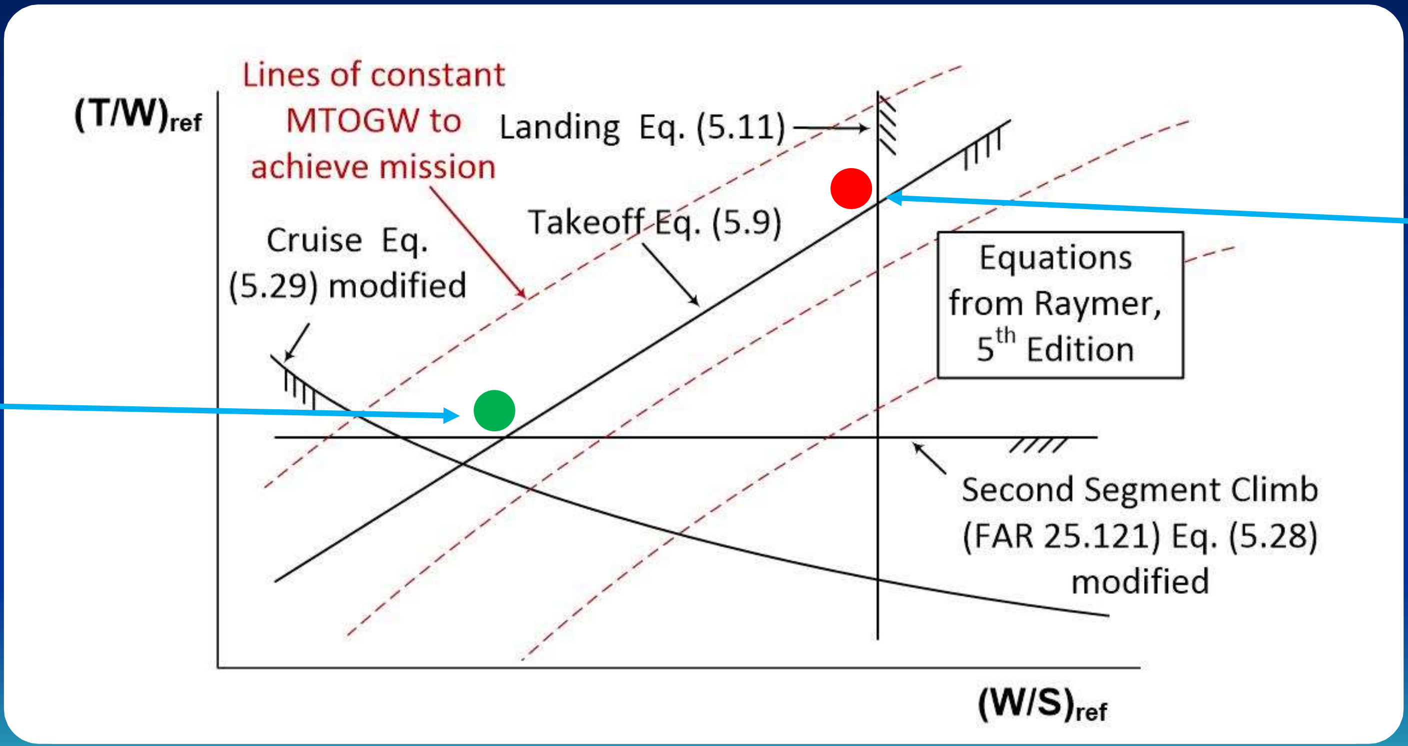
For industrial application, do detailed performance analysis using computer programs



Raymer: Ch 5 T/W vs. W/S
Ch 17 Performance and Flight Mechanics
Ch 19 Trade Studies

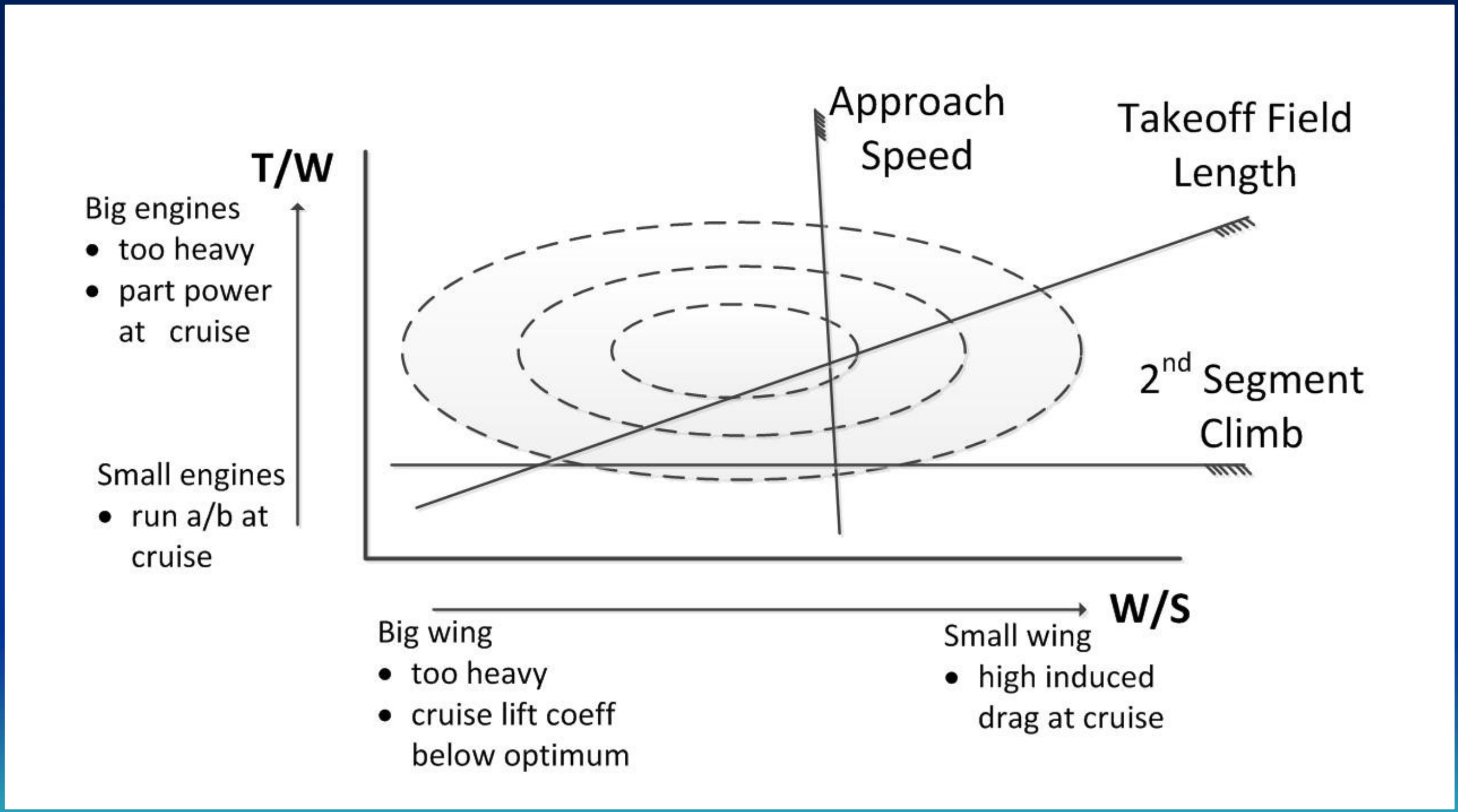
Sizing and Performance – Commercial Aircraft

Select this set of T/W and W/S

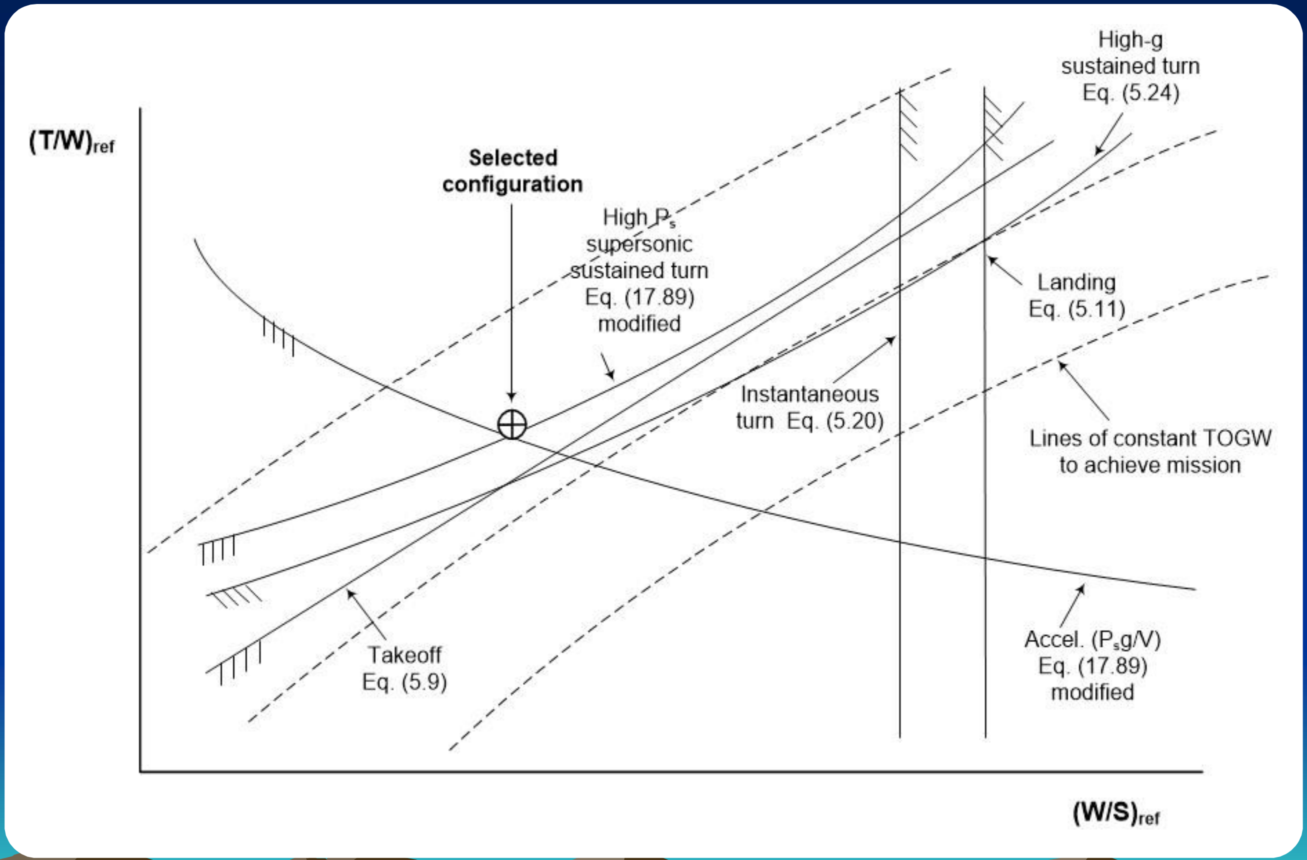


Not this set of T/W and W/S

Knothole Plot for SST with A/B

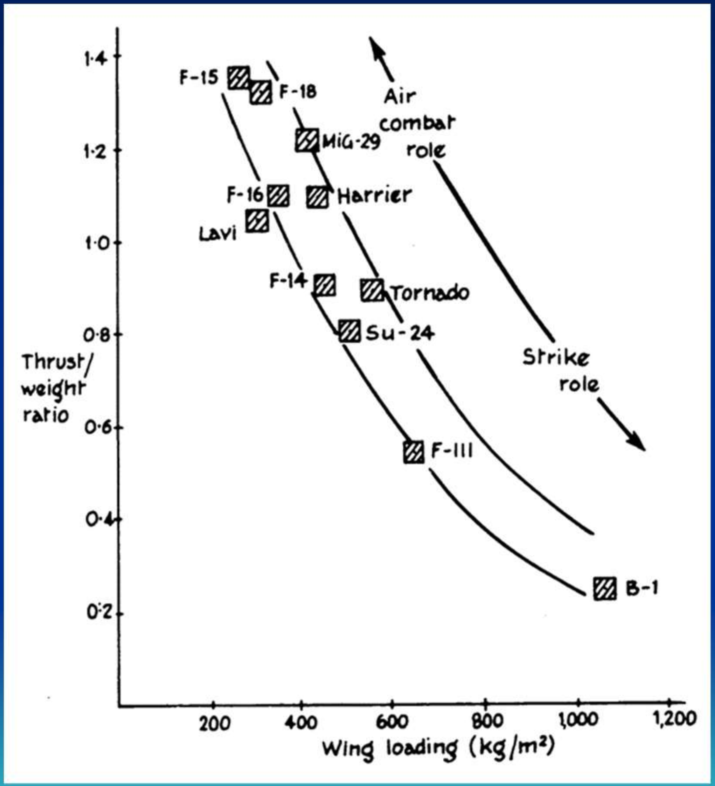


Sizing and Performance – Combat Aircraft



Sizing and Performance – Military Aircraft

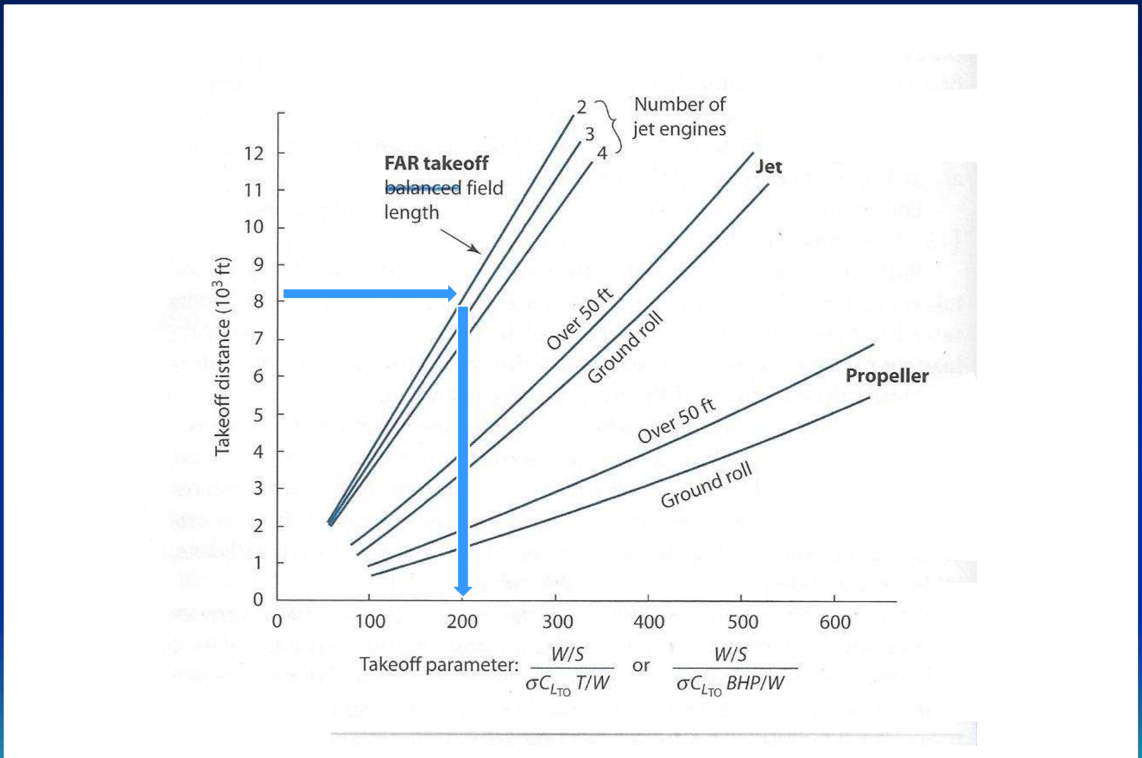
- Air superiority
  - High T/W, low W/S (i.e. big engines, big wing)
- Bomber/strike/interceptor
  - Low T/W, high W/S (i.e. smaller engine, smaller wing)



Takeoff Parameter

Simplified method of estimating T/W = fn(W/S)

Original analysis by Larry Loftin at NASA Langley



Calculation of TOFL Constraint

For known TOFL requirement,
find Takeoff Parameter (TOP) from Fig. 5.4

then $\frac{T}{W} = \frac{\frac{W}{S}}{(TOP)\ \sigma\ C_{L_{TO}}}$

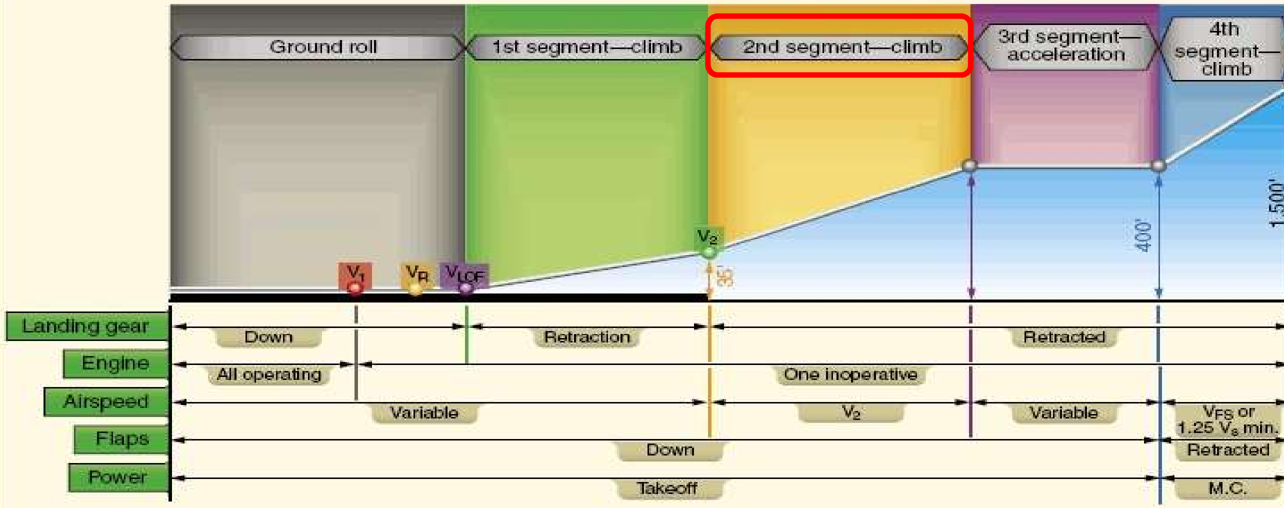
where $\sigma = \frac{\rho}{\rho_0}$

For military jet or propeller aircraft $C_{L_{TO}} = \frac{\left(C_{L_{max}}\right)_{TO}}{1.21}$

For FAR takeoff lines $C_{L_{TO}} = \left(C_{L_{max}}\right)_{TO}$

$\left(\text{Also need} \left(C_{L_{max}}\right)_{TO} \text{ to estimate 2nd segment climb speed, } V_2\right)$

Takeoff Climb Segments

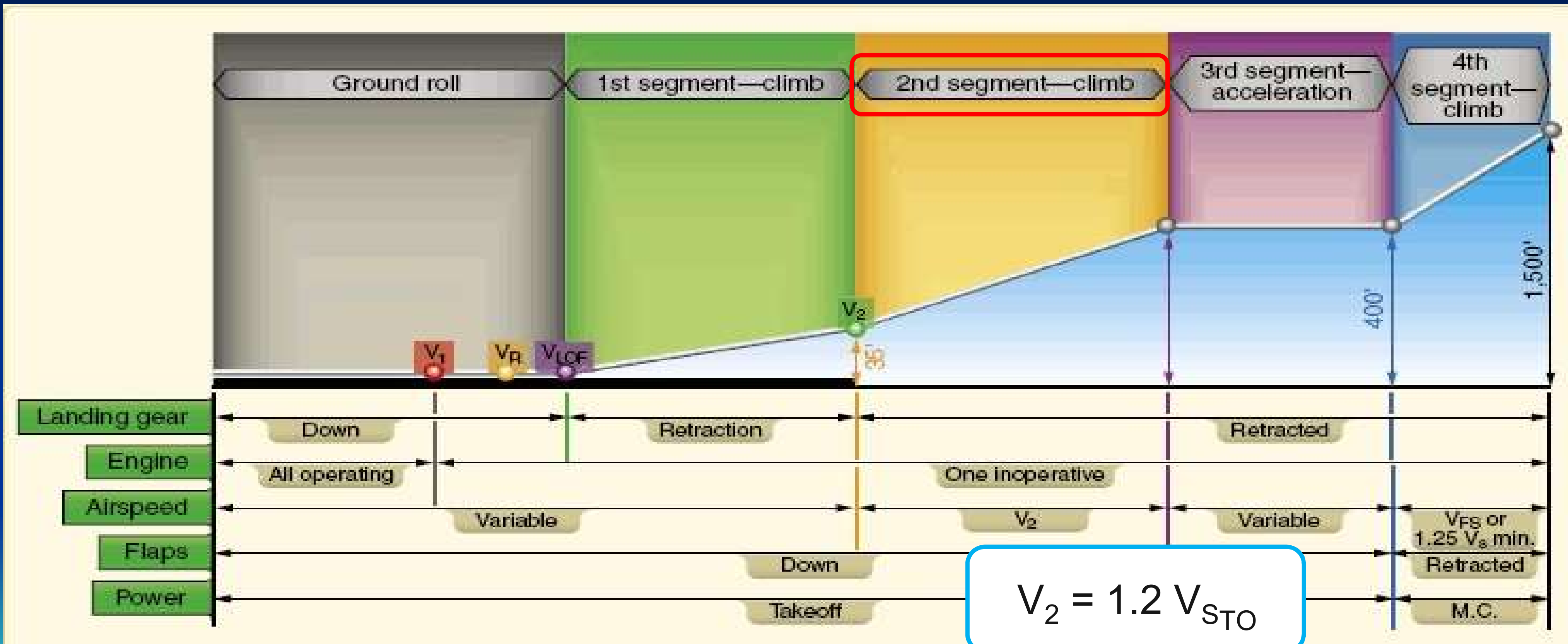


Items		1st T/O segment	2nd T/O segment	Transition (acceleration)	Final T/O segment
*	2 Engine	Positive	2.4%	Positive	1.2%
*	3 Engine	0.3%	2.7%	Positive	1.5%
*	4 Engine	0.5%	3.0%	Positive	1.7%
Wing flaps		T.O.	T.O.	T.O.	Up
Landing gear		Down	Up	Up	Up
Engines		1 Out	1 Out	1 Out	1 Out
Power		T.O.	T.O.	T.O.	M.C.
Air speed		$V_{LOF} \rightarrow V_2$	V_2	$V_2 \rightarrow 1.25\ V_S(\text{Min})$	$1.25\ V_S(\text{Min})$

* Required Absolute Minimum Gradient of Flightpath

M.C. = Maximum continuous
V_1 = Critical-engine-failure speed
V_2 = Takeoff safety speed
V_S = Calibrated stalling speed, or minimum steady flight speed at which the aircraft is controllable
V_R = Speed at which aircraft can start safely raising nose wheel off surface (Rotational Speed)
V_{LOF} = Speed at point where airplane lifts off

Takeoff Climb Segments



$V_2 = 1.2\ V_{STO}$

Takeoff Climb Segments

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Wing flaps		T.O.	T.O.	T.O.	Up
Landing gear		Down	Up	Up	Up
Engines		1 Out	1 Out	1 Out	1 Out
Power		T.O.	T.O.	T.O.	M.C.
Air speed		$V_{LOF} \rightarrow V_2$	V_2	$V_2 \rightarrow 1.25\ V_S(\text{Min})$	$1.25\ V_S(\text{Min})$

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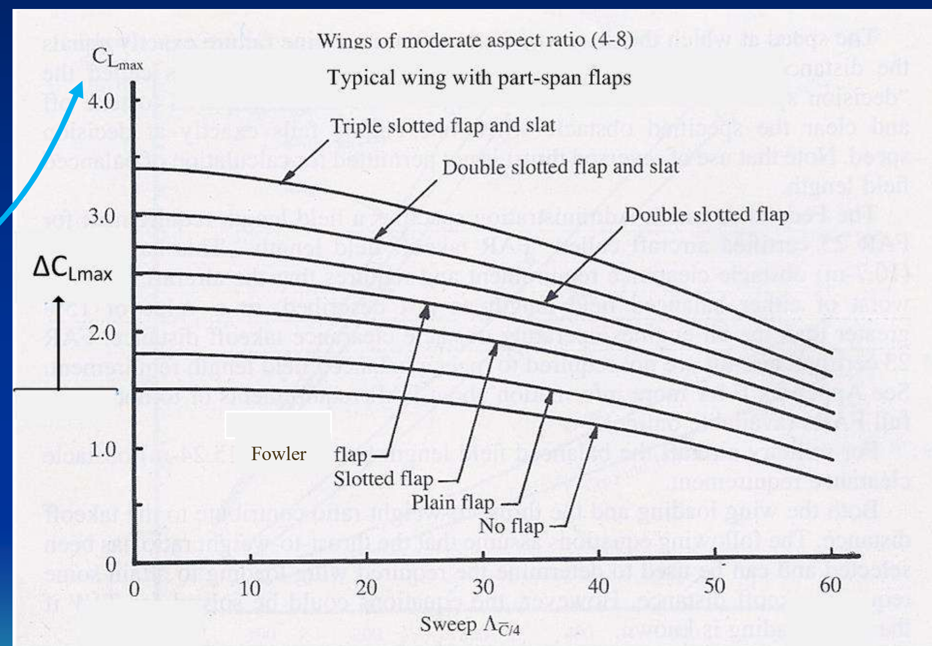
Estimation of Takeoff $C_{L_{max}}$ (Raymer Method)

From thrust lapse rate with speed, we need this to estimate V_S, hence V_2 (=1.2 V_S), and thus thrust at 2nd segment

From Raymer (6th Ed.) page 127 or (5th Ed.) page 128

$$\left(C_{L_{max}}\right)_{TO} \approx 0.8\left(C_{L_{max}}\right)_{Landing}$$

In landing configuration



Raymer Fig. 5.3 Maximum Lift Coefficient

Source: Raymer

Estimation of Takeoff $C_{L_{max}}$ (Torenbeek Method)

If airfoil section is unknown, factor $\Delta C_{L_{max}}$ value from Fig. 5.3

Assume

$$\left(\Delta C_{L_{max}}\right)_{TO} = 0.55\left(\Delta C_{L_{max}}\right)_{Land} \quad \text{(Torenbeek Fig 7.24)}$$

e.g.

from figure on right,

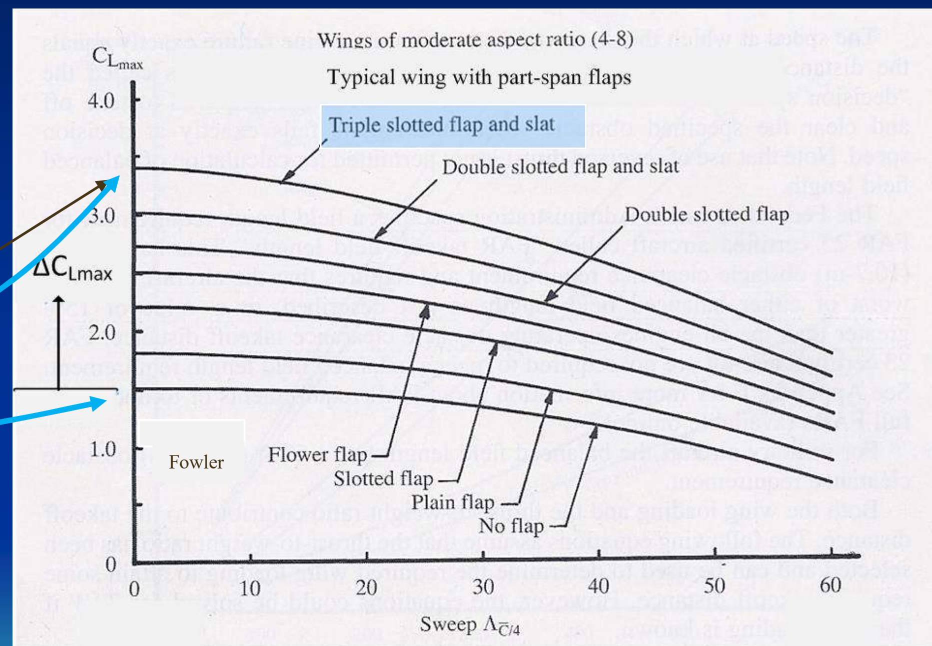
for unswept wing with triple-slotted flap and slat

$$\left(\Delta C_{L_{max}}\right)_{Land} = 3.4 - 1.5 = 1.9$$

so

$$\left(\Delta C_{L_{max}}\right)_{TO} = 0.55 \times 1.9 = 1.05$$

$$\left(C_{L_{max}}\right)_{TO} = 1.5 + 1.05 = 2.55$$



Raymer Fig. 5.3 Maximum Lift Coefficient

Source: Raymer

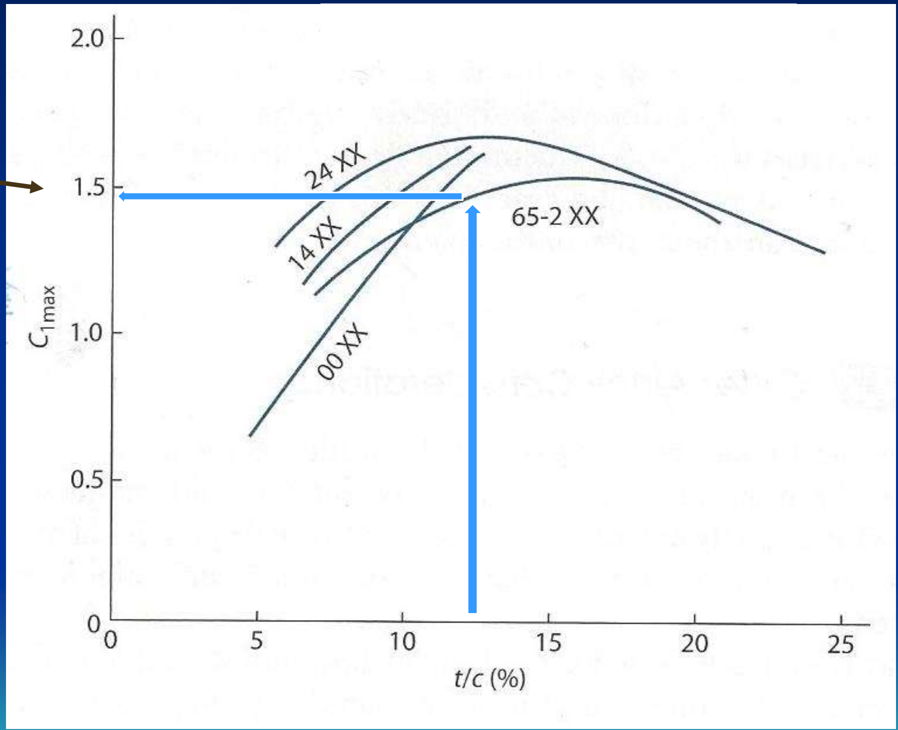
Known Airfoil Section

If airfoil section is known,
get section $C_{l_{max}}$ values from Raymer Fig. 4.13

Use Raymer Eq (5.7) for clean (no flap) curve:

$$C_{L_{max}} = 0.9\, C_{l_{max}} \cos(\Lambda)_{0.25c}$$

Then use $\Delta C_{L_{max}}$ values from Raymer Fig. 5.3 (previous slide)



2^{nd} Segment Climb Estimate of T/W Required

Conditions are

Critical engine inoperative (OEI)

In ground effect

Speed = V_2

Flaps – takeoff

Landing gear – up

Climb gradient (G) (FAR 25.121) :

Two engines – 2.4%

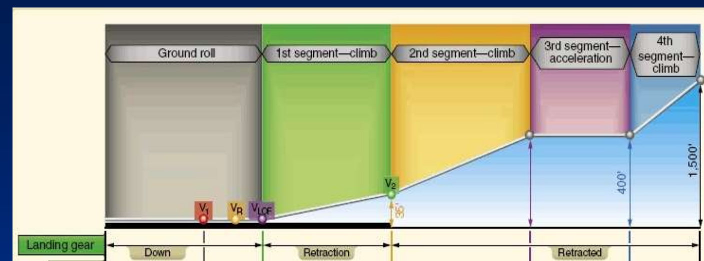
Three engines – 2.7%

Four engines – 3.0%

V_2 is initial climb speed for which

$V_2 > V_{mc}$ (minimum control speed)

$V_2 > 1.2\ V_{s_{TO}}$ (stall speed in takeoff condition)



$$\left(\frac{T}{W}\right)_{2nd\ seg} = \frac{N}{N-1}\left(\frac{1}{\left(\frac{L}{D}\right)_{2nd\ seg}} + G\right)$$

where:

N = number of engines

Assume for first estimate

$$\left(\frac{L}{D}\right)_{2nd\ seg} = 0.75\left(\frac{L}{D}\right)_{max\ clean}$$

Check on this value!

Nominally independent of W/S

$(L/D)_{max\ clean}$ see Raymer Fig. 3.5

Check on L/D for 2nd Segment Climb

Example for 15° flaps

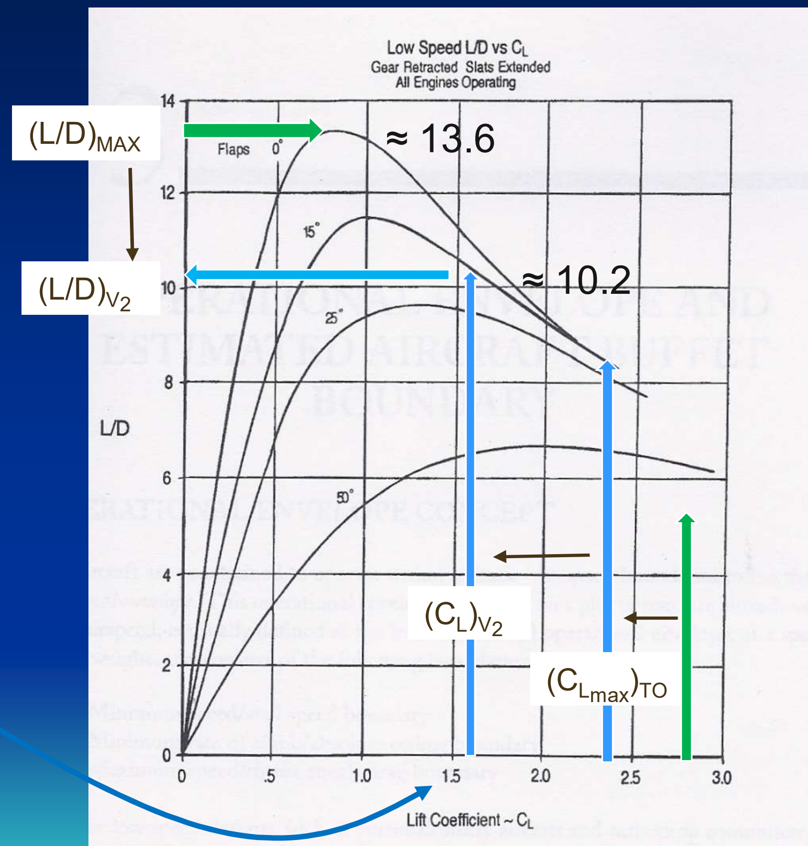
- E.g. for double-slotted flap/slat on wing with $\Lambda_{c/4}$ = 20 deg

  $C_{L_{max}}$ = 2.8 (from Raymer Fig.5.3)
- Assume $(C_{L_{max}})_{V_2} = 0.8\ C_{L_{max}} = 2.24$ (Raymer p.180)

  $V_2 = 1.2V_s$

  $(C_L)_{V_2} = 2.24/1.44 = 1.56$
- Valid assumption that

  $(L/D)_{V_2} \approx 0.75\ (L/D)_{max}$



Source: Schaufele

Calculation of $(T/W)_{2nd\ seg}$

To account for one-engine-inoperative (14 CFR 25.121 (b))

$$\left(\frac{T}{W}\right)_{2nd\ seg} = \frac{N}{N-1}\left(\frac{1}{\left(\frac{L}{D}\right)_{2nd\ seg}} + G\right)$$

Required climb gradient, *G*, is set by FAR requirements

$G = 2.4\%$ for two-engine aircraft

$= 2.7\%$ for three-engine aircraft

$= 3.0\%$ for four-engine aircraft

Finally $$\left(\frac{T}{W}\right)_{ref} = \left(\frac{T}{W}\right)_{2nd\ seg} x \frac{T_{ref}}{T_{2nd\ seg}}$$

Example Calculation of $(T/W)_{2nd\ seg}$

$$\left(\frac{T}{W}\right)_{2nd\ seg} = \frac{N}{N-1}\left(\frac{1}{\left(\frac{L}{D}\right)_{2nd\ seg}} + G\right)$$

For this example:

$N = 2$

$G = 0.024$

$$\left(\frac{L}{D}\right)_{2nd\ seg} = 10.2$$

$$\left(\frac{T}{W}\right)_{2nd\ seg} = 2 \times \left(0.024 + \frac{1}{10.2}\right) = 0.24 \quad \text{at that condition}$$

Finally $\left(\frac{T}{W}\right)_{ref} = \left(\frac{T}{W}\right)_{2nd\ seg} x \ \frac{T_{ref}}{T_{2nd\ seg}}$

Calculation of $(T/W)_{2nd\ seg}$

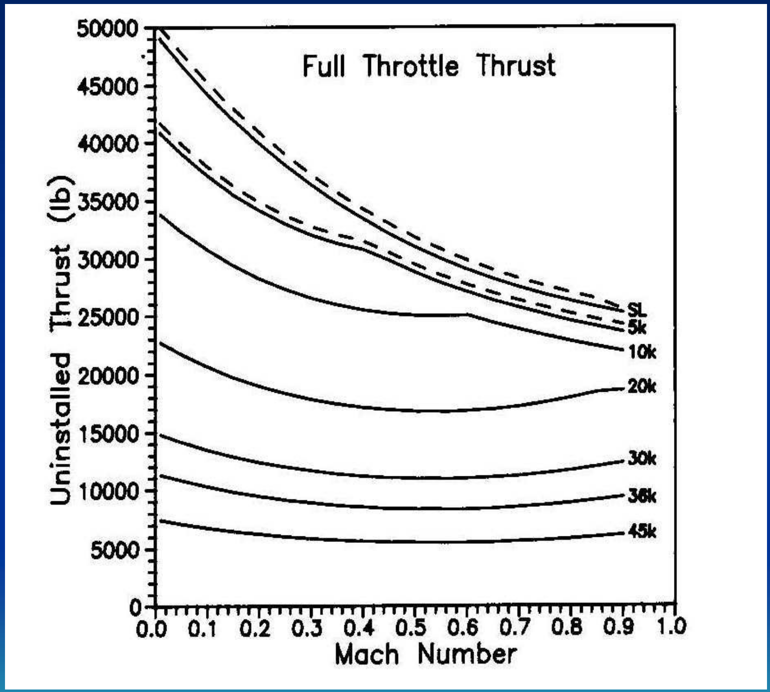
$$\left(C_L\right)_{V_2} = \frac{\left(C_L\right)_{\max TO}}{1.44} = \frac{2.55}{1.44} = 1.77$$

Assume that typical wing loading is $120\ \frac{lb}{ft^2}$

or select anticipated value

So $$q = \frac{\frac{W}{S}}{\left(C_L\right)_{V_2}}) = \frac{120}{1.77} = 68\ \frac{lb}{ft^2}$$

At sea level $$\frac{q}{M^2} = 1481\ \frac{lb}{ft^2}$$



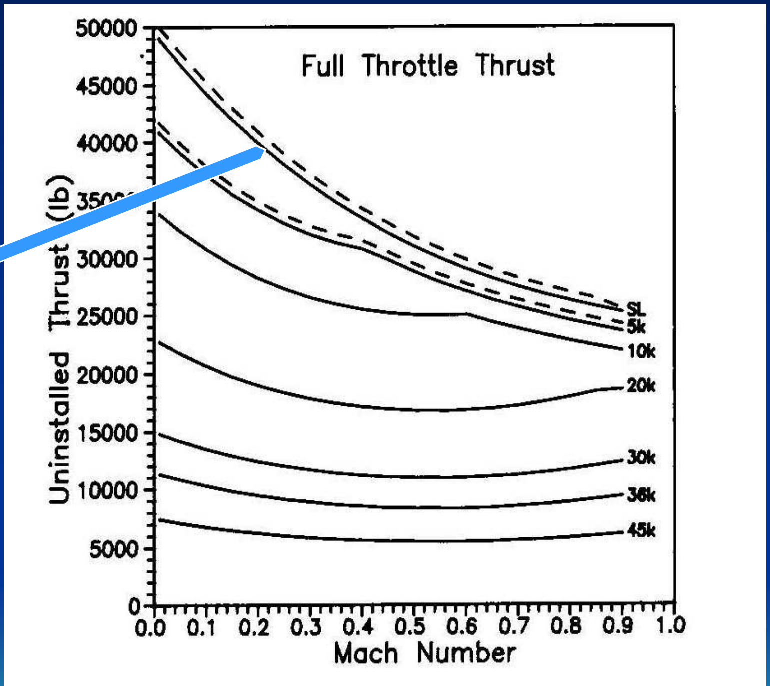
Calculation of $(T/W)_{2nd\ seg}$

So $$M_{2nd\,seg} = \sqrt{\frac{68}{1481}} = 0.21$$

From Raymer Table E.2 for BPR 8 engine at full thrust

$$\frac{T_{M=0.21}}{T_{ref}} = 0.68$$

So $$\left(\frac{T}{W}\right)_{ref} = \left(\frac{T}{W}\right)_{2nd\,seg}\left(\frac{T_{ref}}{T_{2nd\,seg}}\right) = \frac{0.24}{0.68} = 0.35$$



Calculation of Landing Constraint

For specified landing field length (LFL),

$$\left(\frac{W}{S}\right)_{landing} = \sigma C_{L_{max}} \left(\frac{S_{landing}}{80} - S_a\right)$$

FAR 121.195 (b) requires that

$$S_{landing} = 0.6\ LFL$$

otherwise $S_{landing} = LFL$

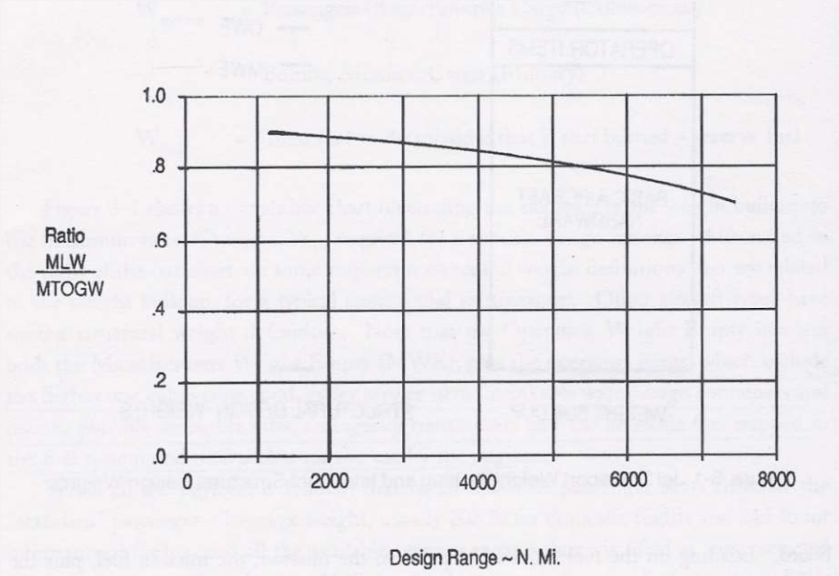
$S_a = 1000$ ft for airliner

$= 600$ ft for GA

$= 450$ ft for STOL with 7 deg glideslope

$C_{L_{max}} =$ maximum lift coefficient on approach

then $\left(\frac{W}{S}\right)_{ref} = \left(\frac{W}{S}\right)_{landing} x \frac{MTOGW}{MLW}$



Source: Schaufele

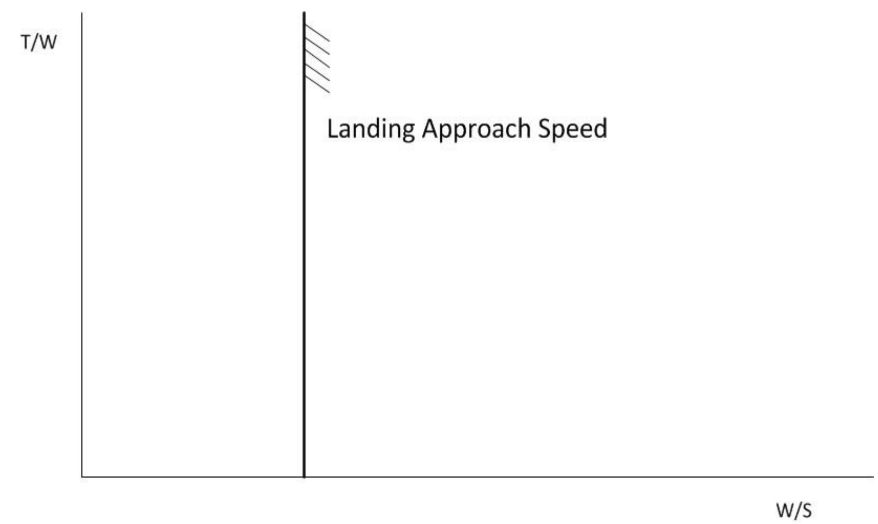
W/S Required for Specified Landing Approach Speed

$$\frac{W}{S} = \frac{1}{2}\rho\left(1.3\,V_{stall}\right)^2\left(C_{L_{max}}\right)_{Land}$$

where

V_{stall} = stall speed in approach condition

$\left(C_{L_{max}}\right)_{Land}$ = maximum C_L in approach condition



Calculation of T/W at start of cruise

At start of cruise (Raymer Eq. (5.29))

$$\frac{T}{W} = \frac{D}{W} + G = \frac{q\,C_{D_0}}{\frac{W}{S}} + \frac{W}{S}\frac{1}{q\pi A\,e} + G$$

$\frac{W}{S}$ is independent variable

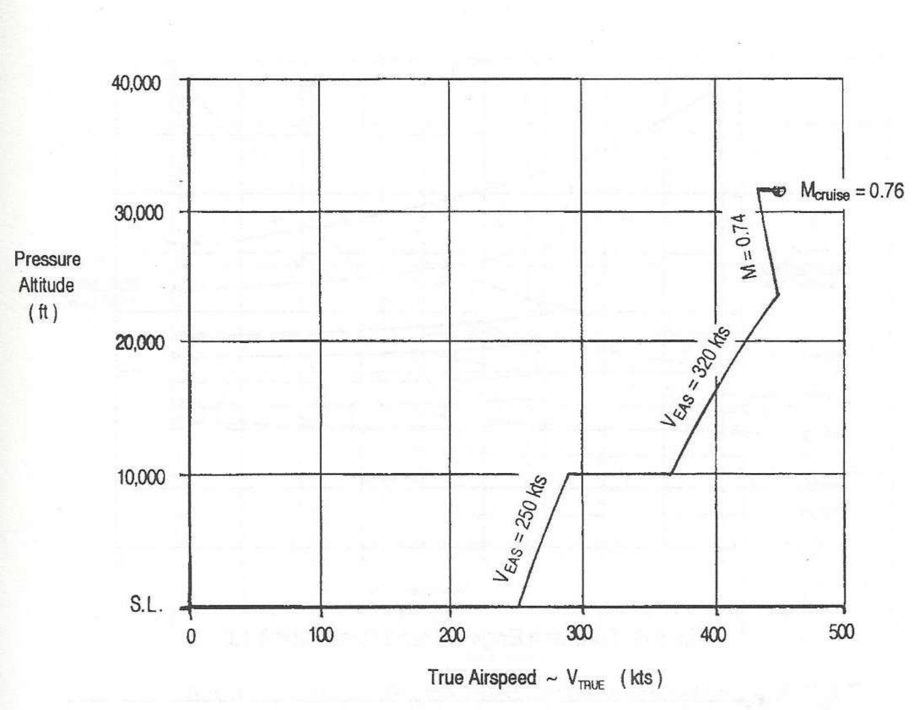
A and q are known (or assumed)

G is defined by FAR (or MilSpec)

$\left(= 300\,\frac{\text{ft}}{\text{min}}\text{ for commercial}\right)$

Estimate C_{D_0}

Assume $e = 0.8$



Equivalent Skin Friction Method

Equivalent Skin Friction Method:

For a flat plate with surface parallel to flow

$$D = C_f \, q \, S$$

where

C_f = skin friction coefficient

S = area

For an airplane

$$D_o = C_{f_e} \, q \, S_{wet}$$

where

C_{f_e} = equivalent skin friction coefficient

S_{wet} = airplane wetted area

$$C_{D_o} = \frac{D_o}{q S_{ref}} = C_{f_e} \frac{S_{wet}}{S_{ref}}$$

Aircraft type	C_{f_e}
Civil transport	0.0026
Bomber	0.0030
Military cargo	0.0035
Air Force fighter	0.0035
Navy fighter	0.0040
Supersonic cruise aircraft	0.0025
Light aircraft - single engine	0.0055
Light aircraft - twin engine	0.0045
Seaplane - propeller driven	0.0065
Seaplane - jet	0.0040

Source: Raymer (with modification)

Climb Requirements

FAR Definition	Climb Rate Requirement [ft/min]
Absolute Ceiling	0
Service Ceiling	100
Operational Ceiling	300

MilSpec Definition	Climb Rate Requirement [ft/min]
Combat Ceiling	500

Thrust/ Weight (T/W) and Wing Loading (W/S)

The End